Metamaterials Mimicking Dynamic Spacetime, D-brane and Noncommutativity in String Theory

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Abstract

We propose a scheme to mimic the expanding cosmos in $1 + 2$ dimensions in laboratory. Furthermore, we develop a general procedure to use nonlinear metamaterials to mimic D-brane and noncommutativity in string theory.

Keywords: Metamaterials; D-brane; Noncommutativity

1. Introduction

String theory is a developing field of modern theoretical physics which attempts to unify quantum mechanics and general relativity. The basic objects in string theory are one dimensional strings and high dimensional D-branes. To be self-consistent, the theory also requires supersymmetry, extra dimensions. The effective geometry on a D-brane becomes noncommutative when there is a nonzero background B field [1]. String theory has made tremendous theoretical progresses in the past 15 years, such as explaining microscopic origin of the black hole entropy [2] in certain cases, the discovery of AdS/CFT [3]. However, string theory remains hard to be tested by experiments due to the extremely high energy scale.

It will be of great interest if one can mimic some phenomena of string theory in laboratory. Fortunately, the developments of metamaterials and transformation optics [4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15] may be helpful for us to achieve this goal. Recently, metamaterials were used to design various interesting devices such as electromagnetic cloak [16, 17], perfect lens [18], illusion devices[19, 20] and so on. They can even be used to make an artificial

black hole [21, 22, 23] and to mimic cosmos [24, 25]. For example, in recent works [24, 25], we find that the Casimir energy of the electromagnetic field in de Sitter space is proportional to the size of the horizon, the same form of the holographic dark energy [26]. We suggest to make metamaterials to mimic static de Sitter space in laboratory and measure the predicted Casimir energy.

In this paper we move on and propose to use metameterials to mimic some phenomena in cosmology and string theory. We design an approach to mimic a dynamic space-time in $1+2$ dimensions by materials with constant permeability and varying permittivity, to avoid the complex issue of designing metameterials with synchronously varying permittivity and permeability [27]. Our proposal can be implemented by 3-level atoms or a nonlinear dielectric. Besides, we further suggest to use nonlinear metamaterials to mimic D-brane and noncommutative geometry in string theory. This topic is scientifically interesting, since it implies the possibility of testing and investigating some aspects of string theory at low energy scale in laboratory. Besides, the various properties of D-brane, noncommutativity and nonlinear electrodynamics will motivate us to design interesting optical devices.

This paper is organized as follows. In Sec. II, we develop an executable experimental procedure to mimic expanding cosmos in $1+2$ dimensions with abundant interesting effects such as redshift, the cosmic microwave background (CMB) and the future event horizon. In Sec. III, we develop in theory a general procedure to use metamaterials to mimic gravity, D-brane, noncommutativity and so on. We conclude in Sec. IV.

2. Mimic dynamic spacetime in 1+2 dimension

In this section we discuss the issue of mimicking a $1+2$ dimensional dynamic space-time in metameterials. To do this let us consider the analogy between curved spacetime and dielectric medium for electromagnetic fields, as was explicitly studied in [10, 16]. Especially, it is proven that light has the same behavior in dielectric medium and curved spacetime, under the conditions that the permittivity and the permeability are related to the metric by

$$\varepsilon^{ij} = \frac{\varepsilon_0\sqrt{-g}}{-g_{00}}g^{ij},\ \mu^{ij} = \frac{\sqrt{-g}}{-\varepsilon_0 c^2 g_{00}}g^{ij}\ . \tag{1}$$

It should be stressed that this equation is valid only in 1+3 dimensions. This relation can be used to design metamaterials and mimic specific kind

of spacetime. For example, to mimic an expanding universe, the authors of [27] proposed to vary the permeability by split-ring resonators, and change the permittivity by electrooptical modulation.

Theoretically, one can make use of Eq.(1) to mimic an arbitrary spacetime. However, to mimic a dynamic spacetime, for instance, an expanding universe, one should simultaneously control the permeability and the permittivity to ensure them varying synchronously. Unfortunately, this is a hard task for experimental physics. To avoid this problem in this paper we just consider the case of mimicking a 1+2 dimensional spacetime. In this section we study two models, including a real 1+2 dimensional spacetime and a 1+2 dimensional subspace of the 1+3 dimensional spacetime. In both cases we find that the permittivity and the permeability can be independent of each other, and the above problem is automatically resolved. Besides, it is natural and useful to consider the simplified case of realizing the 1+2 dimensional spacetime before turning to the much more complicated case of realizing a 1+3 dimensional spacetime. The metric of a homogeneous and isotropic universe in 1+2 dimensions takes the form

$$ds^2 = g_{00}c^2dt^2 + g_{11}dx^2 + g_{22}dy^2 \ , \tag{2}$$

with $g_{11} = g_{22}$. Using the method developed in the next section we find that the dielectric with the following permeability and permittivity can be used to mimic this 1+2 dimensional metric

$$\mu = \frac{\sqrt{-g}}{-\varepsilon_0 c^2 g_{00}}, \quad \varepsilon^{ij} = \frac{\varepsilon_0\sqrt{-g}}{-g_{00}} g^{ij} \ (i,j = 1,2) \ . \tag{3}$$

It is clear that the permittivity and the permeability are independent of each other. This remarkable property can be used to simplify experimental design. In fact, to mimic the expanding cosmos, one can keep the permeability as a constant and only vary the permittivity. We rewrite the metric in terms of the permittivity and the permeability from Eq.(3)

$$ds^2 = -\frac{\varepsilon_0^2}{\varepsilon^2(t)}c^2dt^2 + \frac{\mu(t)\varepsilon_0^2c^2}{\varepsilon(t)}(dx^2 + dy^2) \ . \tag{4}$$

To realize the Friedmann-Robertson-Walker(FRW) metric

$$ds^2 = -c^2d\tau^2 + a^2(\tau)(dx^2 + dy^2) \ , \tag{5}$$

we redefine the time coordinate as $d\tau = \varepsilon_0 dt/\varepsilon(t)$. Correspondingly we should express all the parameters in terms of this new time τ rather than the laboratory time t. For example, $\mu(\tau) = 1/(\varepsilon_0 c^2)$ and $\varepsilon(\tau) = \varepsilon_0/a^2(\tau)$.

Note that the Maxwell theory in 1+2 dimensions can be derived from that in 1+3 dimensions by imposing the self-consistent conditions $E_3 = B^1 = B^2 = 0$ and $\partial_3 E_1 = \partial_3 E_2 = \partial_3 B^3 = 0$. Hence, we can model this 1+2 dimensional metric in dielectric with translation invariance along the z-axis and use transverse cross section perpendicular to the z-axis to mimic the two-dimensional space we are interested in. One may polarize photons to keep the magnetic field along z-axis, so that the magnetic field acts as a scalar on the cross section. Meanwhile, to confine them completely on the cross section, it is required that photons have no momentum along the axis. Therefore, one can make an analogy between the electromagnetic fields on the cross section and the counterparts in 1+2 dimensional spacetime

$$\tilde{E}_i \sim E_i \ (i = 1, 2), \quad \tilde{B}_3 \sim B \ . \tag{6}$$

Accordingly, one can also make an analogy between the relevant parameters of the dielectric and those in 1+2 dimensions

$$\tilde{\varepsilon}^{ij} \sim \varepsilon^{ij} \ (i, j = 1, 2), \quad \tilde{\mu}^{33} \sim \mu \ . \tag{7}$$

Thus, to mimic the metric (5), one only need to design metameterials with the property

$$\tilde{\varepsilon}^{ij} \ = \varepsilon_0 a^{-2}(\tau) \ \delta^{ij} \ \ (i, j = 1, 2), \quad \tilde{\mu}^{33} = 1/(\varepsilon_0 c^2) \ . \tag{8}$$

We can also make the electric field parallel to the z-axis so that the electric field becomes a scalar on the cross section. In this case we can not mimic the Maxwell theory in 1+2 dimensions in which electric fields are vectors. Instead, we can use it to mimic the Maxwell theory in a 1+2 dimensional subspace of the 1+3 dimensional spacetime. As in the above case, to be confined on the cross section, photons should have no momentum along the z-axis. When electric field is polarized along the axis, the nonzero components of the electromagnetic fields are E_3 and B_{ij} $(i, j = 1, 2)$, and the relevant dielectric parameters are ε^{33} and μ^{ij} $(i, j = 1, 2)$. It should be stressed that if we consider only the 1+2 dimensional subspace, the irrelevant dielectric parameters such as ε^{ij} and u^{33} need not take the exact form (1). In fact, they can take arbitrary forms and do not affect the polarized photons

on the cross section. Setting $g_{33} = 1$, one can derive the relevant dielectric parameters from Eq.(1) as

$$\varepsilon^{33} = \frac{\varepsilon_0\sqrt{-g}}{-g_{00}} \ , \quad \mu^{ij} = \frac{\sqrt{-g}}{-\varepsilon_0 c^2 g_{00}} g^{ij} \ (i,j=1,2) \ . \tag{9}$$

We have the metric of the 1+2 dimensional subspace from Eq.(9)

$$ds^2 = -\frac{1}{\varepsilon_0^2 c^2 \mu^2(t)} dt^2 + \frac{\varepsilon^{33}(t)}{\varepsilon_0^2 c^2 \mu(t)} (dx^2 + dy^2) \ . \tag{10}$$

Next we set $\mu = 1/(\varepsilon_0 c^2)$ and denote $\varepsilon^{33}(t)$ by $\varepsilon(t)$, it follows that

$$ds^2 = -c^2 dt^2 + \frac{\varepsilon(t)}{\varepsilon_0}(dx^2 + dy^2) \ . \tag{11}$$

where t is just the laboratory time. The scale factor, which signifies the scale of the universe, is $a(t) = \sqrt{\varepsilon(t)/\varepsilon_0}$. Now we get the FRW metric in 1+2 dimensional flat universe. One can mimic an expanding universe using metameterials with increasing $\varepsilon(t)$.

In the rest of this section, we focus on the experimental design (11) with the electric field polarized along the axis. Experimentally one has several alternative methods to vary the permittivity, e.g., by making use of the electrooptical effect and the model consisting of N identical 3-level atoms [28] (the second method allows a comparably wider range of variation). Many interesting effects, such as the future event horizon, redshift and the varying CMB temperature, are expected in the experiment.

If the permittivity increases fast enough, a future event horizon will arise

$$r_h(t) = a(t)\int_t^\infty \frac{cdt'}{a(t')} = \sqrt{\varepsilon(t)}\int_t^\infty \frac{cdt'}{\sqrt{\varepsilon(t')}} \ . \tag{12}$$

This is the largest distance that photons can travel. In the case of inflation, $a(t) = e^{Ht}$ (H is the Hubble constant), and $r_h(t) = cH^{-1}$. Thus cH^{-1} is the limited distance light rays can propagate (the optical distance, not the coordinate distance).

For light with frequency ν_0 emitted at time t_0, its frequency at time t_1 takes the form [29]

$$\nu_1/\nu_0 = a(t_0)/a(t_1) = \sqrt{\varepsilon(t_0)/\varepsilon(t_1)} \ . \tag{13}$$

If one increases $\varepsilon(t)$, the frequency decreases accordingly and light is red-shifted. The number density of photons is described by the Bose-Einsein distribution

$$n(\nu,t)d\nu = \frac{8\pi\nu^2 d\nu/c^3}{\exp(h\nu/k_{\mathcal{B}}T(t)) - 1} \ . \tag{14}$$

Integrating this equation, one finds that the total photon number and the total energy $N \propto T^3$ and $U \propto T^4$, respectively. Thus the average energy of photons takes the form $\bar{E} = h\bar{\nu} = U/N \propto T$. Notice that the temperature varies as $T \propto \bar{\nu} \propto 1/a(t) \propto 1/\sqrt{\varepsilon(t)}$, which is exactly the same form of the CMB temperature.

3. Mimic D-brane and Noncommutativity in String Theory

In this section we demonstrate the feasibility of mimicking generalized electrodynamics by metamaterials. Let us start with the Maxwell equations in a dielectric medium

$$\partial_i B^i = 0, \quad \epsilon^{ijk}\partial_j E_k + \partial_t B^i = 0 \ , \tag{15}$$

$$\partial_i D^i = 0, \quad \epsilon^{ijk}\partial_j H_k - \partial_t D^i = 0 \ . \tag{16}$$

where the electric displacement D^i and magnetizing field H_i are related to the electric field E_i and the magnetic field B^i through

$$D^i = \varepsilon^{ij}E_j, \quad B^i = \mu^{ij}H_j. \tag{17}$$

For generalized electrodynamic theory in an arbitrary background spacetime, the Lagrangian takes the form $L(F_{\mu\nu}, g_{\mu\nu})$. From the definition of the field strength $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, one can derive the Bianchi identity

$$\partial_\mu F_{\nu\rho} + \partial_\nu F_{\rho\mu} + \partial_\rho F_{\mu\nu} = 0 \ . \tag{18}$$

By defining

$$E_i = F_{i0} \ , \quad B^i = \frac{1}{2c}\epsilon^{ijk}F_{jk} \ , \tag{19}$$

one can rewrite the Bianchi identity Eq.(18) as Eq.(15). From the equations of motion (Eq.(44) in the Appendix), one can derive Eq.(16) as long as the electric displacement and magnetic field are defined by

$$D^i = \frac{\partial L(F_{\mu\nu}, g_{\mu\nu})}{\partial E_i}, \quad H_i = -\frac{\partial L(F_{\mu\nu}, g_{\mu\nu})}{\partial B^i}. \tag{20}$$

For details of the derivation please refer to the Appendix. Clearly, in the framework of the generalized electrodynamical theory, photons have the same behavior as those in the dielectric medium satisfying the constitutive relations (20), due to the same expression of Maxwell equations in these two cases. This similarity implies a promising method to mimic generalized electrodynamics, D-brane and noncommutativity in string theory.

Following our procedure, the well-known analogy (1) between curved spacetime and dielectric can be easily derived. For simplicity, here we take the case of 1+2 dimensions as an example. The Lagrangian of the electromagnetic field in curved space is

$$L(F) = -\frac{1}{4}\varepsilon_0\sqrt{-g}F^2 = -\frac{1}{4}\varepsilon_0\sqrt{-g}g^{\mu\alpha}g^{\nu\beta}F_{\mu\nu}F_{\alpha\beta}\ , \tag{21}$$

from which one can derive

$$D^i = \frac{\partial L}{\partial E_i} = \frac{\partial L}{\partial F_{i0}} = \frac{\varepsilon_0\sqrt{-g}}{-g_{00}}g^{ij}E_j\ , \tag{22}$$

$$H = -\frac{\partial L}{\partial B} = c\frac{\partial L}{\partial F_{21}} = \frac{-\varepsilon_0 c^2 g_{00}}{\sqrt{-g}}B\ . \tag{23}$$

Comparing the above equations with constitutive relations (17), one gets

$$\varepsilon^{ij} = \frac{\varepsilon_0\sqrt{-g}}{-g_{00}}g^{ij}, \quad \mu = \frac{\sqrt{-g}}{-\varepsilon_0 c^2 g_{00}}\ . \tag{24}$$

With this procedure at hand, we are in a position to investigate some interesting examples. In the following paragraphs, we study the mimicking of single D_p-brane and noncommutativity in string theory.

In string theory, the low-energy dynamics of single D_p-brane is described by the Born-Infeld action

$$S = -\varepsilon_0 T_{D_p}\int d^{p+1}x\sqrt{-\det(\eta_{\mu\nu} + 2\pi\alpha' F_{\mu\nu})}\ , \tag{25}$$

where T_{D_p} is the tension, $p+1$ is the number of dimensions of the world-volume. From the above arguments, we can use an appropriate dielectric to mimic the low-energy dynamics of D_p-brane. Take $p = 2$ as an example, the corresponding dielectric has the following permittivity and permeability

$$\varepsilon = \frac{4\varepsilon_0 T_{D_p}\pi^2\alpha'^2}{\sqrt{1 + 4\pi^2\alpha'^2(c^2B^2 - \vec{E}^2)}} = \mu^{-1}/c^2\ . \tag{26}$$

As in the case of 1+2 dimensional expanding universe, one can mimic this D_2-brane by dielectric with translation invariance along the z-axis and impose the self-consistent conditions $E_3 = B^1 = B^2 = 0$, $\partial_3 E_1 = \partial_3 E_2 = \partial_3 B^3 = 0$.

We encourage experimentalists to design such nonlinear metamaterials, they have great applications in mimicking string theory in laboratory. In such nonlinear metamaterials, a mass of novel phenomena are expected. For example, there is a upper limit of electric field intensity, the self-energy of the charge is finite, there is no birefringence and so on. To keep the permittivity and the permeability (26) real, it is clear that $E \leq 1/(2\pi\alpha')$. In general, when these parameters become complex, the system will exhibit an instability with abundant photons production and absoption. There is a correspondence of such instability in string theory. As we know, D-branes are objects where open strings can end, and charges carried by the two ends of a string are opposite. Thus, for an open string with both ends on the D-brane, an electric field will pull the two endpoints of string apart. If E is large enough, it will overcome the string tension to tear apart the string. It implies an instability similar to that in the dielectric. This instability of D-brane is hard to be observed in laboratory, but it is possible to mimic this effect by metamterials. Besides, metamaterials seem to have the unique capability to mimic such novel phenomenons.

In string theory the effective geometry becomes noncommutative (NC) when there is a background B-field [1]

$$[x^\mu, x^\nu] = i\ \theta^{\mu\nu}\ , \tag{27}$$

where x^μ are spacetime coordinates and $\theta_{\mu\nu}$ is a constant antisymmetric tensor. The generalized Yang-Mills action in NC space is

$$S = -\frac{1}{4}\varepsilon_0 \int d^4x\ \hat{F}_{\mu\nu} * \hat{F}^{\mu\nu}\ , \tag{28}$$

where the product $*$ and field strength $F_{\mu\nu}$ are defined as

$$f(x) * g(x) = exp(\frac{i}{2}\theta^{\mu\nu}\frac{\partial}{\partial\xi^\mu}\frac{\partial}{\partial\zeta^\nu})f(x+\xi)g(x+\zeta)|_{\xi=\zeta=0}\ , \tag{29}$$

$$\hat{F}_{\mu\nu} = \partial_\mu\hat{A}_\nu - \partial_\nu\hat{A}_\mu - i[\hat{A}_\mu, \hat{A}_\nu]_*\ . \tag{30}$$

In general, the NC Yang-Mill theory can be converted into a commutative but nonlinear counterpart by the Seiberg-Witten map. For example, when

$\hat{F}_{\mu\nu}$ is a constant, the Seiberg-Witten map is

$$\hat{F}_{\mu\nu} = \frac{F_{\mu\nu}}{1 + \sqrt{\frac{\varepsilon_0}{\hbar c}} F^{\alpha\beta}\theta_{\alpha\beta}} \quad , \tag{31}$$

where $F_{\alpha\beta}$ is the ordinary field strength in commutative space. From the procedure developed in this section, one can mimic the NC Maxwell theory by metamaterials satisfying

$$\begin{aligned}
D^i &= \frac{\varepsilon_0 E^i}{(1 + 2\sqrt{\frac{\varepsilon_0}{\hbar c}} E_l \theta^{l0} + \epsilon_{lmn}\sqrt{\frac{c\varepsilon_0}{\hbar}}\theta^{lm} B^n)^2} \\
&+ \frac{2\sqrt{\frac{\varepsilon_0}{\hbar c}}\varepsilon_0 (c^2 \vec{B}^2 - \vec{E}^2)\theta^{i0}}{(1 + 2\sqrt{\frac{\varepsilon_0}{\hbar c}} E_l \theta^{l0} + \epsilon_{lmn}\sqrt{\frac{c\varepsilon_0}{\hbar}}\theta^{lm} B^n)^3} \ , \\
H_i &= \frac{\varepsilon_0 c^2 B_i}{(1 + 2\sqrt{\frac{\varepsilon_0}{\hbar c}} E_l \theta^{l0} + \epsilon_{lmn}\sqrt{\frac{c\varepsilon_0}{\hbar}}\theta^{lm} B^n)^2} \\
&- \varepsilon_0 c \frac{(c^2 \vec{B}^2 - \vec{E}^2)\epsilon_{ijk}\sqrt{\frac{\varepsilon_0}{\hbar c}}\theta^{jk}}{(1 + 2\sqrt{\frac{\varepsilon_0}{\hbar c}} E_l \theta^{l0} + \epsilon_{lmn}\sqrt{\frac{c\varepsilon_0}{\hbar}}\theta^{lm} B^n)^3} \ .
\end{aligned} \tag{32}$$

In NC geometry, Lorentz symmetry is broken. This can lead to birefringence and direction dependent shift in the speed of light. For simplicity, we consider a constant electromagnetic field with small fluctuations, thus the Seiberg-Witten map (31) for constant field strength is valid. Replace $F_{\mu\nu}$ with $f_{\mu\nu} + F_{\mu\nu}$, where $f_{\mu\nu}$ denotes the constant background field and $F_{\mu\nu}$ is the small fluctuation. The resulting Lagrangian reduces to

$$L = -\frac{\varepsilon_0}{4(1 + \sqrt{\frac{\varepsilon_0}{\hbar c}}\theta^{\mu\nu} f_{\mu\nu})^2} F_{\mu\nu}F^{\mu\nu} - \frac{\varepsilon_0}{4(1 + \sqrt{\frac{\varepsilon_0}{\hbar c}}\theta^{\mu\nu} f_{\mu\nu})^2} k_{\kappa\lambda\mu\nu} F^{\kappa\lambda} F^{\mu\nu} \ , \tag{33}$$

where

$$k_{\alpha\beta\gamma\delta} = \frac{3\frac{\varepsilon_0}{\hbar c} f_{\mu\nu} f^{\mu\nu}\theta_{\alpha\beta}\theta_{\gamma\delta}}{(1 + \sqrt{\frac{\varepsilon_0}{\hbar c}}\theta^{\mu\nu} f_{\mu\nu})^2} - \frac{2\sqrt{\frac{\varepsilon_0}{\hbar c}}(f_{\alpha\beta}\theta_{\gamma\delta} + \theta_{\alpha\beta} f_{\gamma\delta})}{1 + \sqrt{\frac{\varepsilon_0}{\hbar c}}\theta^{\mu\nu} f_{\mu\nu}} \ , \tag{34}$$

we keep only terms up to the second order in $F_{\mu\nu}$. The linear terms are absent due to the equations of motion of the background field. The equation of motion of the fluctuation is

$$\partial^\nu F_{\mu\nu} + k_{\mu\nu\rho\sigma}\partial^\nu F^{\rho\sigma} = 0 \ . \tag{35}$$

For a plane wave $F_{\mu\nu}(x) = F_{\mu\nu}(p)e^{-ip_\alpha x^\alpha/\hbar}$, from the above equation one can get

$$M^{ij}E_j = (-\delta^{ij}p^2 - p^j p^k - 2k^{i\mu\nu j}p_\mu p_\nu)E_j = 0 \ . \tag{36}$$

Following the general method, by requiring the determinant of M^{ij} vanishes, one can derive the dispersion relation of photons, then obtain the group velocity [30]

$$\upsilon_g = c|\nabla_{\vec{p}}p^0| = c(1+\rho\pm\sigma) \ . \tag{37}$$

to leading order in $k_{\mu\nu\rho\sigma}$, where

$$\rho = -\frac{1}{2}\tilde{k}^\alpha_\alpha, \quad \sigma^2 = \frac{1}{2}(\tilde{k}_{\alpha\beta})^2 - \rho^2 \ , \tag{38}$$

with

$$\tilde{k}^{\alpha\beta} = k^{\alpha\mu\beta\nu}\hat{p}_\mu\hat{p}_\nu, \quad \hat{p}^\mu = p^\mu/|\vec{p}| \ . \tag{39}$$

As mentioned above, the velocity of light is direction dependent. Furthermore, since two solutions exist for $\vec{E}$, it implies the birefringence effect, where these two components of light propagate independently with different velocities.

To end this section, we list the main results in the leading order in the NC parameters and study an interesting optical phenomenon in such a NC space. It is likely that photons will spend less time traveling along polyline than along straight line. For simplicity, we focus on the case that all NC parameters vanish, other than

$\theta^{03} = \theta > 0$. To leading order of θ, the constitutive relations (32) of metamaterials become,

$$\begin{aligned} H_i &= \varepsilon_0 c^2(1+4\sqrt{\frac{\varepsilon_0}{\hbar c}}\theta E_3)B_i, \\ D^i &= \varepsilon_0(1+4\sqrt{\frac{\varepsilon_0}{\hbar c}}\theta E_3)E^i - 2\varepsilon_0(c^2\vec{B}^2 - \vec{E}^2)\sqrt{\frac{\varepsilon_0}{\hbar c}}\theta\delta^i_3 \ . \end{aligned} \tag{40}$$

For simplicity, we consider only the case $\vec{E} = 0$, $\vec{B} = (0,0,B)$ for the background field. Focus on the first order velocity (37), from Eq.(34) and Eq.(39) we learn that only $\hat{p}^\mu$ in the leading order of θ contribute, thus we can take $\hat{p}^\mu = (1, \sin\varphi, 0, \cos\varphi)$ in x-z plane as a good approximation ($\varphi \in [0, \pi/2]$ is the angle between the momentum of photons and background magnetic field). The velocity of photons then becomes,

$$\upsilon_g = c \pm 2c\sqrt{\frac{c\varepsilon_0}{\hbar}}\theta B\sqrt{(1+\cos^2\varphi)\sin^2\varphi} \ . \tag{41}$$

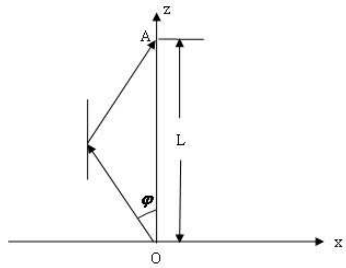

Figure 1: The polyline and straight line between O and A.

We focus on the case $\upsilon_g = c + 2c\sqrt{\frac{c\varepsilon_0}{\hbar}}\theta B\sqrt{(1+\cos^2\varphi)\sin^2\varphi}$ below. It is clear that this velocity increases monotonically with $\varphi \in [0, \pi/2]$, thus the velocity assumes its minimum value $\upsilon_g = c$ when photons propagate along the direction of the magnetic field. It should be stressed that though the group velocity (41) generally exceeds the speed of light c in vacuum, it does not mean that information can transfer faster than c. Various experiments have verified that it is possible for the group velocity of light in materials significantly exceed the speed of light in vacuum. However, in all these cases, the speed of information remains less than or equal to c [31].

As depicted in Fig.1, the distance between these two points O and A is L. The time that photons will spend to cover this distance along the straight line is

$$t_0 = L/c \; . \tag{42}$$

Photons can also travel along another path, that is, the polyline depicted in Fig.1. In this case, photons are reflected to point A by a mirror, and the time it will take to travel along this polyline is

$$t_1 = \frac{L/\cos\varphi}{c + 2c\sqrt{\frac{c\varepsilon_0}{\hbar}}\theta B\sqrt{(1+\cos^2\varphi)\sin^2\varphi}} \; . \tag{43}$$

One can always find a nonempty interval for φ to make sure $t_1 < t_0$. For instance, set $\sqrt{\frac{c\varepsilon_0}{\hbar}}\theta B = 0.1$, one can find ct_1/L varies with φ as depicted in Fig.2. Hence, it takes less time for photons to move along the polyline than that along the straight line for $\varphi \in (0, 0.47)$. And the fastest path for photons to travel to point A is along the angle $\varphi = 0.24$. Thus, the least time for photons to travel from point O to point A is $t_1 = 0.97L/c$.

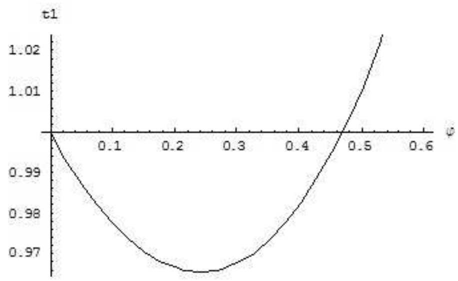

Figure 2: The time photons spend along the polyline with respect to the direction of propagation.

It should be stressed that in such a NC space with a constant background magnetic field, free photons still travel along straight line, but the fastest path for light to propagate between two points is generally a polyline. This interesting, counter-intuitive effect is due to Lorentz violation, which is believed to serve as signals of new physics coming from the Planck scale. Since the direct detection of Lorentz violation is difficult for the time being, it is of great significance if one can mimic and detect this effect in laboratory.

4. Conclusion

In this paper we discuss the issue of using metameterials to mimic some phenomena in cosmology and string theory. We propose a simple procedure to mimic a $1 + 2$ dimensional expanding universe. We find that this can be implemented by materials with constant permeability and varying permittivity, so that the troublesome problem of designing metameterials with synchronously varying permittivity and permeability is automatically avoided. Moreover, we further suggest to use nonlinear metamaterials to mimic D-brane and noncommutative geometry in string theory. The permittivity and permeability of the corresponding metamaterials are calculated and provided in Sec. III. We see that it is possible to test and investigate some aspects of string theory at low energy scale in laboratory. However, it should be stressed that our proposals are as yet in theory, the metamaterials which we suggest to mimic D-brane and noncommutative geometry are extremely difficult to make. We hope that with the progress in the field of metamaterials our proposals will be realized in the future.

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Appendix

In this appendix, we show that with definition (20), the Maxwell equations in generalized electrodynamic theory and dielectric coincide.

For a generalized electrodynamic theory, the Euler-Lagrange equation takes the form

$$\partial_\mu \frac{\partial L}{\partial(\partial_\mu A_\nu)} - \frac{\partial L}{\partial A_\nu} = 0 \ . \tag{44}$$

Note that

$$D^i = \frac{\partial L}{\partial E_i} = \frac{\partial L}{\partial(\partial_i A_0)} \ , \qquad \frac{\partial L}{\partial A_0} = \frac{\partial L}{\partial(\partial_0 A_0)} = 0 \ , \tag{45}$$

one can easily obtain

$$\partial_i D^i = \partial_i \frac{\partial L}{\partial(\partial_i A_0)} = 0 \ . \tag{46}$$

Similarly, note that

$$H_k = -\frac{\partial L}{\partial B^k} = -\frac{1}{2} c \epsilon_{ijk} \frac{\partial L}{\partial F_{ij}} \ , \tag{47}$$

$$\epsilon^{mnk} H_k = -\frac{c}{2} \epsilon^{mnk} \epsilon_{ijk} \frac{\partial L}{\partial F_{ij}} = -c \frac{\partial L}{\partial F_{mn}} = \frac{c}{2} \frac{\partial L}{\partial(\partial_n A_m)} \ , \tag{48}$$

from the Euler-Lagrange equation, one can derive

$$\epsilon^{mnk} \partial_n H_k = \frac{c}{2} \partial_n \frac{\partial L}{\partial \partial_n A_m} = -\frac{c}{2} \partial_0 \frac{\partial L}{\partial \partial_0 A_m} = c \partial_0 \frac{\partial L}{\partial E_m} = \partial_t D^m \ . \tag{49}$$

Finally, with Eq.(46) and Eq.(49), we get the same Maxwell equations (16) as in dielectrics. It should be mentioned that a similar transformation-optical equivalence relations has been derived in [32].

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